Conclusions

The utilization of the second harmonic of the amplitude-modulated signal for the control of the search oscillator enables shifting the microwave oscillator frequency close enough to the cavity resonant frequency for the automatic frequency locking circuit to start the control. This makes it possible to increase the range of thickness and permittivity of the dielectric films measured by the system.

Acknowledgment

The authors wish to thank Dr. S. S. Stuchly for many profitable discussions throughout this project.

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Anomalous Convergence of Iterative Methods in the Numerical Solution of Electromagnetic Problems

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***Abstract*—Iterative methods applied to eigenvalue equations can lead to anomalous convergence. It is shown that this can occur when some of the eigenvalues are complex and the corresponding eigenvectors satisfy a particular condition. A method of distinguishing between anomalous and effective convergence is indicated.**

Many electromagnetic problems can be solved by the finite difference technique, which leads to a system of difference equations whose coefficient matrix A is generally real. The approximate solution of the continuous problem is then obtained by solving the matrix eigenvalue problem $(A-\lambda I)x=0$. If the matrix A is very large it is necessary to use an iterative method for the computation of the eigenvalues.

A useful procedure [1] that allows the computation of all the real eigenvalues of A is to introduce the semidefinite positive matrix

$$C(\lambda) = (A - \lambda I)^T(A - \lambda I) \tag{1}$$

and then to compute an eigenvalue λ in the following manner.

1) Equation $C(\lambda^{(0)})x=0$ ($\lambda^{(0)}$ being a guess at λ) is solved by an iterative method, e.g., successive displacements, starting with an arbitrary real vector $x^{(0)}$.

2) A reestimate $\lambda^{(1)}$ is computed from the Rayleigh quotient and the value obtained replaces $\lambda^{(0)}$ in C.

3) An alternation between steps 1 and 2 is carried out until $\lambda^{(i)}$ seems to have converged to λ.

The described procedure generally works satisfactorily; however, we verified that anomalous convergences can occur when the matrix A is unsymmetric. It is the purpose of this short paper to discuss these anomalies and to show how to identify them.

Let us refer to a very simple example. Consider the 5×5 circulant matrix [2] whose first row is

$$a_{1j} = [0, 1, 2, 3, 6] \tag{2}$$

and use it as a test matrix.

By using as iterative method in step 1 the method of successive displacements and continuing the procedure for calculating λ until $\lambda^{(i)}$ and $\lambda^{(i+1)}$ differ by less than 0.1 percent, we obtained the following results.

$\lambda^{(0)}$	$x^{(0)}$	(Convergence Value) λ_{conv}
−3	1, 2, 3, 4, 5	−4.115
3	1, 2, 1, 2, 1	−4.115
5	1, 2, 1, 2, 1	12

It is easy to verify that only $\lambda_{\text{conv}}=12$ is an eigenvalue of (2), while $\lambda_{\text{conv}}=-4.115$ is an anomalous convergence value. However, it can be noticed that the value −4.115 corresponds to the real part of the complex eigenvalues of A. The behavior shown in the example has also been found in solving other real matrices obtained from electromagnetic problems using the finite difference technique. The anomalous convergence values are always found to be the real part of complex eigenvalues. The above result must, therefore, be borne in mind in the case of problems that are not schematized by a symmetric matrix, since, as complex eigenvalues may exist, there is the possibility that the iterative procedure may give rise to a convergence towards a quantity that does not correspond to an actual eigenvalue. We will, therefore, examine in detail the whole procedure for studying in which cases an anomalous convergence may occur.

Let $\mathcal{L}(C, \lambda^{(0)})$ be the error-reducing (or iteration) matrix [2] relative to the iterative method adopted to solve $C(\lambda)x=0$, and let μ_i and I_i be the eigenvalues and the corresponding eigenvectors of $\mathcal{L}$. Both μ_i and I_i are in general complex. Supposing that the matrix A is not defective [3], the initial real vector $x^{(0)}$ can be expressed as

$$x^{(0)} = \sum_{i=1}^{n} (\alpha_i I_{ir} + \beta_i I_{ij}) \tag{3}$$

where α_i and β_i are real constants and $I_i = I_{ir} + jI_{ij}$. Performing s iterations we obtain

$$x^{(s)} = \sum_{i=1}^{n} (\alpha_i \mathcal{L}^s I_{ir} + \beta_i \mathcal{L}^s I_{ij}) \tag{4}$$

where a generic eigenvalue and the corresponding eigenvector are related by

$$\begin{aligned} \mathcal{L}^s I_r &= |\mu|^s (I_r \cos s\theta - I_j \sin s\theta) \\ \mathcal{L}^s I_j &= |\mu|^s (I_j \cos s\theta + I_r \sin s\theta) \end{aligned} \tag{5}$$

with $\mu = |\mu| e^{j\theta}$.

Let μ_1 be the eigenvalue having the greatest absolute value. Let us distinguish the two cases

1) μ_1 real ($I_{1r} = I_1$; $I_{1j} = 0$).

Equation (4) for s increasing tends to

$$x^{(s)} = \alpha_1 \mu_1^s I_1.$$

Therefore, $x^{(s)}$ tends to assume the form of I_1 and the Rayleigh quotient tends to become constant. Moreover, the Rayleigh quotient gives a value $\lambda^{(i+1)}$ of λ that is closer than $\lambda^{(i)}$ to a true eigenvalue.

2) μ_1 complex.

Since the matrix $\mathcal{L}$ is real, $\mu_2 = \mu_1^*$ and $I_2 = I_1^*$. Taking account of (5), it is found that, as s increases, $x^{(s)}$ tends to

$$x^{(s)} = (\alpha_1 + \alpha_2)\mathcal{L}^s I_{1r} + (\beta_1 - \beta_2)\mathcal{L}^s I_{1j} = a(s) I_{1r} + b(s) I_{1j} \tag{6}$$

where $a(s)$ and $b(s)$ are oscillating functions of s. Consequently, $x^{(s)}$ oscillates in sign as s varies. However, let us consider the Rayleigh quotient

$$\frac{x^{(s)T} A x^{(s)}}{x^{(s)T} x^{(s)}} = \frac{(aI_{1r}^T + bI_{1j}^T) A (aI_{1r} + bI_{1j})}{(aI_{1r}^T + bI_{1j}^T)(aI_{1r} + bI_{1j})}.$$

The quotient, which will generally vary with a and b (i.e., with s),

Manuscript received December 6, 1971; revised February 24, 1972. This work was supported by Consiglio Nazionale delle Ricerche, Italy.
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may assume a constant value, independently of s. This occurs if the following relations hold:

$$\frac{I_{1r}^T A I_{1r}}{I_{1r}^T I_{1r}} = \frac{I_{1j}^T A I_{1j}}{I_{1j}^T I_{1j}} = \frac{I_{1r}^T A I_{1j} + I_{1j}^T A I_{1r}}{I_{1r}^T I_{1j} + I_{1j}^T I_{1r}} = k. \quad (7)$$

If we now indicate by $\lambda_r + j\lambda_j$ the eigenvalues of A, and by $u+jv$ the corresponding eigenvectors, it can be shown that, in order that all the relations (7) may hold, it is necessary, for an eigenvector of A (which we indicate by $\bar{u}+j\bar{v}$), that

$$\bar{u}^T\bar{v} = 0 \quad \text{and} \quad |\bar{u}| = |\bar{v}|. \quad (8)$$

Moreover, if (8) holds, we have

$$k = \bar{\lambda}_r. \quad (9)$$

We can conclude that if, on increasing s, the Rayleigh quotient tends to a constant value k, one of the following conditions holds: 1) k is closer than $\lambda^{(i)}$ to a true eigenvalue; 2) k is equal to the real part of a complex eigenvalue.

To verify whether the convergence is anomalous or not, the form of $x^{(s)}$ must be examined. If $x^{(s)}$, as s increases, oscillates in sign, the convergence is anomalous and, in order to obviate this inconvenience, the procedure must be reinitiated after changing $\lambda^{(0)}$ or $x^{(0)}$.

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Letters

Comments on "Noise in IMPATT Diode Amplifiers and Oscillators"

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In the above paper,[1] an extensive contribution on the noise of IMPATT oscillators was presented, which is based on the theory of Kurokawa [1]. In our opinion, however, the theoretical results of this work do not apply in all cases to the experiments reported. The authors define a load angle θ (15a) which appears in the equations for the AM and FM noise [(16b) and (17b)] and the correlation coefficient [(18b). Eqs. (16b), (17b), and (18b)] describe the noise of the current through the diode, which is of minor interest and difficult to measure. The AM noise of the load current, which in reality was measured,[1] differs considerably from the noise of the diode current if $\theta \neq 90°$. In this case, an additional FM–AM conversion term appears which, under certain conditions, cancels exactly the load-angle-dependent terms in the expression for the AM noise of the diode current. These conditions are as follows.

1) The transforming network between the diode current and the load current is lossless but otherwise arbitrary; losses in series or parallel to the load or to the active device, however, are allowed.

2) The oscillator is tuned to maximum output power.

Then the AM noise and the correlation coefficient of the load current are independent of θ, while the FM noise remains as in (17b). This can be seen from the following formulas where the symbols of Thaler *et al.*[1] have been used:

$$S_{AA}^0(\Omega) = S_{cc}(\Omega)/A_0^2 \cdot \left|\frac{\partial Z_D}{\partial A}\right|^2 \cdot \cos^2 \eta$$

$$S_{\omega\omega}^0(\Omega) = S_{cc}(\Omega)/A_0^2 \cdot \left|\frac{\partial Z_L}{\partial \omega}\right|^2 \cdot \sin^2 (\theta - \eta)$$

$$\gamma_{A\omega}^0 = -\sin \eta.$$

If the conditions 1) and 2) are not exactly satisfied, S_{AA}^0 and $\gamma_{A\omega}^0$ will show only a weak dependence upon θ as derived in [2]. The equations given above have been found to be in close agreement with experimental results obtained for cavity-stabilized Gunn oscillators. In these experiments the distance of the matched cavity from the Gunn diode was varied by a set of disks in order to vary θ without changing the optimum operating conditions. However, the resonance frequency of the cavity has been varied,[1] which introduces a certain mismatch outside the center frequency. Then a discussion of the experimental results becomes more difficult because the tuning condition 2) is violated. The application of (16b) and (18b) is also in this case not justified.

Manuscript received September 22, 1971.
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Reply[2] by Hans-Jörg Thaler, Gerhard Ulrich, and Gerhard Weidmann[3]

The comments of Schiek and Schünemann on the results presented in our paper[1] are based on a misinterpretation of the equivalent circuit used in our analysis. This can be shown by analyzing the oscillator circuit in a reference plane at the load terminals. In this representation the operating point of the oscillator in the complex impedance plane is given by the lines of the frequency and RF current dependence of the active device impedance intersecting each other in the point of the real load impedance. The impedance transformation from the reference plane at the active device terminals (used in our original analysis) to the new reference plane does not change the intersection angles of the two loci which enter the formulas for the oscillator noise spectra. Therefore, the load current and the loop current in our equivalent circuit have the same dependence on the oscillator parameters, especially on the load angle θ. The experimental results presented in our paper clearly contradict the state-

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